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A conjecture on a quantum limit of semiconductor lasers

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A relationship between the maximum operating temperature of semiconductor lasers and their emission wavelength is conjectured. The conjecture is supported by a wide variety of existing experimental data for visible and infrared double heterojunction and quantum well lasers, quantum cascade lasers, as well as more esoteric devices such as Type-II antimonide-based mid-infrared and p -Ge and impurity-based Terahertz devices. The relationship developed may enable the ultimate performance of mid- and far-infrared, as well as Terahertz semiconductor lasers to be predicted.

The Terahertz region of the spectrum[1] remained the last region of the electromagnetic spectrum to be explored for one simple reason: it lies at the junction between the electronic technologies of increasingly higher frequency current (electron) oscillations in millimetre wave devices and the longer wavelength transitions between discrete states which characterise infrared optical technologies. Subsequently the Terahertz region of the spectrum lies at the boundary between classical and quantum physics.

Semiconductor light emitting diodes and lasers produce quanta of electromagnetic radiation (photons) of energy $h\nu$ through the transitions of electrons between states, where h is Planck's constant and ν is the frequency of the electromagnetic radiation. In quantum dot[2] and impurity-based devices[3] these states are discrete, however in the vast majority of devices these quantised states broaden into bands—a continuous range of energies between two fixed limits. In contrast to gas lasers and masers which can produce microwave to optical frequencies at room temperature, solid state lasers have the parallel mechanism of non-radiative phonon emission (lattice vibrations) competing against photon (light) emission. Again it is another point of interest that Terahertz represents the range of frequencies at which the photon energies $h\nu$ are of the same order as the phonon energies $\hbar\omega$ ($\hbar = h/(2\pi)$ and ω is the angular frequency of the lattice vibration). The competing non-radiative phonon emission rate is dependent on $N_0 + 1$, where the phonon occupation number N_0 is given by:

$$N_0 = \frac{1}{\exp(\hbar\omega/kT) - 1} \quad (1)$$

with kT representing the thermal energy density within the crystal lattice.

Until recently there has been no need to try and link the energies $h\nu$, $\hbar\omega$ and kT together, but the impetus driving the development of mid-infrared[4] and Terahertz quantum cascade lasers[5] has begun to bridge the gap between the classical and quantum worlds and the questions ‘*What is the ultimate long wavelength limit?*’ and ‘*What are the maximum operating temperatures of Terahertz quantum cascade lasers?*’ have started to arise more frequently.

There are complicated approaches which could be followed to try to answer these questions, for example, one could use

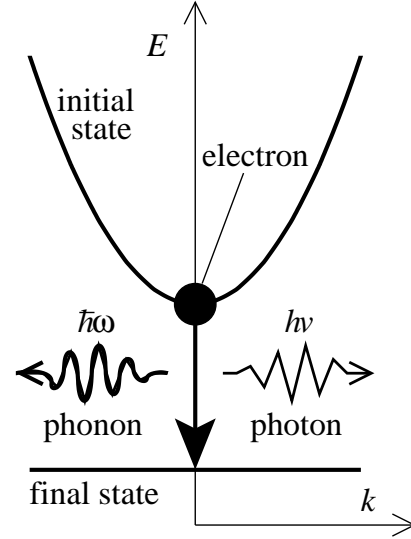


FIG. 1: Schematic diagram illustrating the competition between radiative (photon-generating $h\nu$) and non-radiative (phonon-generating $\hbar\omega$) transitions.

a physical model of quantum cascade lasers[6, 7] and could do lengthy calculations of the gain versus current profile for designs for a large number of wavelengths. Alternatively one could resort to experimental data and derive an empirical relationship. But neither of these methods is quick, the results would not be that transparent and doubts would remain over generality to different material systems and different device designs.

It is clear that when the phonon energy $\hbar\omega$ is equal to the energy separation between the quantised energy levels (and hence the photon energy $h\nu$) that it might be expected that the detrimental phonon emission process will begin to be significant. However, at very low temperatures ($kT \ll \hbar\omega$, see equation 1) there are few phonons, with these processes perhaps becoming significant when the thermal energy is of the order of the phonon energy, i.e. $kT = \hbar\omega$, see Fig. 1. Thus in the search for a relationship between the emission wavelength λ of a semiconductor laser and its maximum operating temperature T , one is guided by two arguments to suggest

that favourable conditions for sustaining a population inversion and achieving lasing occur when:

$$h\nu > \hbar\omega \quad \text{and} \quad kT < \hbar\omega \quad (2)$$

which can be combined to give:

$$h\nu > kT \quad \text{or} \quad \frac{hc}{\lambda} > kT \quad (3)$$

where c is the speed of light. This relationship looks too simple, so it is important to look around for any supporting evidence and this is provided by the data in Fig. 2 which plots the maximum operating temperature against wavelength for state-of-the-art quantum cascade lasers in both the InGaAs/AlInAs on InP and the GaAs/AlGaAs on GaAs systems for pulsed and continuous wave (*cw*) operation. The figure also shows the ‘limit’ predicted by equation 3 as a solid line. The surprising point to note is that all experimental data to date obeys the limit.

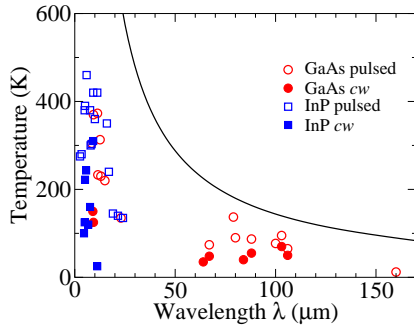


FIG. 2: The points represent the highest operating temperature as a function of emission wavelength for state-of-the-art quantum cascade lasers as reported in the literature[8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41]. The solid line represents the speculated ‘quantum limit’ $h\nu = kT$.

There is nothing special about quantum cascade lasers and so it is worthwhile seeing if experimental data from other types of semiconductor laser conform to this speculated limit.

The type-II interband cascade lasers have operating wavelengths in the short-wave (3–5 μm) infrared and are now approaching room temperature[42, 43, 44] and the lead salt lasers have emission wavelengths from 3–20 μm and *cw* operation above 200 K[45] which puts both of these categories below the solid line and in the bottom left hand corner of Fig. 2. Looking back at Fig. 2 it is clear that it is in the Terahertz region of the spectrum where the quantum cascade lasers come closest to reaching the limit of equation 3, so it is important to look at other types of Terahertz semiconductor laser. Bründermann *et al.*[46] report on a range of *p*-Ge Terahertz lasers which can be tuned from 1 to 4 THz (300–75 μm). In these devices the holes are accelerated by an electric field under the influence of an external magnetic field, Bründermann reports maximum operating temperatures up to 36 K and hence these devices also obey the limit (Note at a wavelength of 300 μm , equation 3 would imply a temperature of 47 K). It is this class of devices which is likely to challenge the limit most strongly with Komiyama *et al.*[47] reporting emission at 1.8 mm at 4.2 K (equation 3 suggests a maximum operating temperature for lasing as 7 K). Even quantum-dot-like devices such as those which use internal impurity transitions to generate Terahertz radiation, see for example [48, 49], are limited to low temperature operation. In the same vein, the application of a magnetic field along the growth axis of semiconductor heterostructures leads to the in-plane localisation of charge carriers and hence the discretisation of the quantum well subbands. In the case of quantum cascade lasers this has led to the so-called ‘quantum box cascade lasers’[50], but investigations of these devices have been focussed on reducing the threshold current or increasing the luminescence intensity[51, 52, 53] or inducing lasing that doesn’t otherwise exist[54] and not on increasing the operating temperature or wavelength.

In summary, a simple relationship which links the maximum operating temperature of a semiconductor laser with its emission wavelength has been proposed. A great deal of experimental data has been cited which supports this ‘quantum limit’, however as the limit has been argued and not rigorously proven, it can only have the status of a conjecture which it is hoped will stimulate discussion.

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